

Summary of §30 Munkers

Codes: 1 = first countable, 2 = second countable, C = compact, L = Lindelöf, M = metrizable, S = separable (countable dense subset).

Basic relationships.

M	\implies	1	pg 130-1.
$C\&M$	\implies	2	Exercise 4.
2	\implies	1	obvious.
2	\implies	L	Thm 30.3.
2	\implies	S	Thm 30.3.
$M\&S$	\implies	2	Exercise 5a.
$M\&L$	\implies	2	Exercise 5b.

Thus for metrizable spaces $L \Leftrightarrow 2 \Leftrightarrow S$.

Inherited by Subspaces?

1 - yes (Thm 30.2).

2 - yes (Thm 30.2).

L - no (Example 5), but yes if closed (Exercise 9).

S - no (Exercise 9), but yes if open (Willard, Thm 16.4b).

Preserved by countable Products?

1 - yes (Thm 30.2).

2 - yes (Thm 30.2).

L - no, not even for finite (Example 4), but $L \times C$ is L (Exercise 14).

S - yes (Exercise 14).

Images preserved?

Cont image of L is L (Exercise 11).

Cont image of S is S (Exercise 11).

Cont open image of 1 is 1 (Exercise 12).

Cont open image of 2 is 2 (Exercise 12).