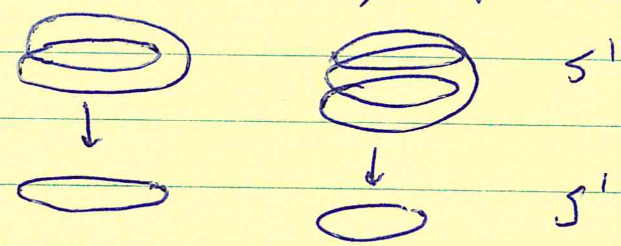
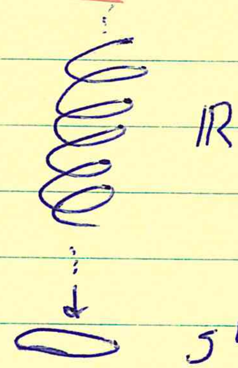
§53 Covering Spaces

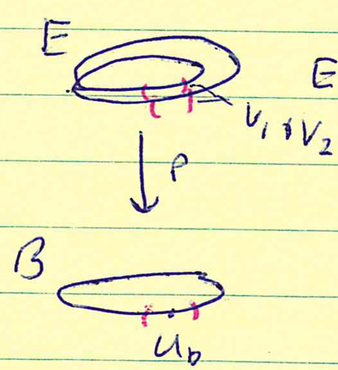
Intuitive Examples Hair band as double, triple covering space of S^1



Phone cord: Infinite, $\mathbb{R}$ as covering space of S^1



Def Let $p: E \to B$ be cont. onto map. If $\forall b \in B$, $\exists\, U_b$, nbhd of b, s.t. $p^{-1}(U_b)$ is a disjoint union of open sets $\{V_\alpha\}$ s.t. $\forall \alpha$, $p|V_\alpha$ is a homeo taking V_α to U_b, then p is a covering map, E is a covering space of the base space B and U_b is a evenly covered subset of B.



E = covering sp.

p = covering map

B = base space

U_b is evenly covered.

Ex Let $p: \mathbb{R} \to S^1$ be given by $p(x) = (\cos(2\pi x), \sin(2\pi x))$. This is a covering map (phone cord). The text proves this as Thm 53.1. Study it carefully. In some sense it is <u>the</u> cannonical example of a covering map.

Facts Covering spaces and maps behave nicely wrt subspaces and finite products.

- Let $p: E \to B$ be a covering map, ~~and~~ $B_0 \subset B$, and $E_0 = p^{-1}(B_0)$. Then $p|E_0 : E_0 \to B_0$ is a covering map. See Thm 53.2.

- Let $p_i : E_i \to B_i$, $i = 1, 2$, be covering maps. Then $p_1 \times p_2 : E_1 \times E_2 \to B_1 \times B_2$ is a covering map. See Thm 53.3.

Note Later ((Ch 13) we will find a relationship between $\pi_1(E)$ and $\pi_1(B)$.

Ex Using the second fact (53.3) we can construct an infinite-to-one covering map from $\mathbb{R}^2$ to the torus: $p: \mathbb{R}^2 \to S^1 \times S^1$. See Example 4 is textbook.