Summary of §30 Munkers

Codes: 1 = first countable, 2 = second countable, C = compact, L = Lindelöf, M = metrizable, S = separable (countable dense subset).

Basic relationships.

M	$\Longrightarrow$	1	pg 130-1.
$C\&M$	$\Longrightarrow$	2	Exercise 4.
2	$\Longrightarrow$	1	obvious.
2	$\Longrightarrow$	L	Thm 30.3.
2	$\Longrightarrow$	S	Thm 30.3.
$M\&S$	$\Longrightarrow$	2	Exercise 5a.
$M\&L$	$\Longrightarrow$	2	Exercise 5b.

Thus for metrizable spaces $L \Leftrightarrow 2 \Leftrightarrow S$.

Inherited by Subspaces?

1 - yes (Thm 30.2).
2 - yes (Thm 30.2).
L - no (Example 5), but yes if closed (Exercise 9).
S - no (Exercise 9), but yes if open (Willard, Thm 16.4b).

Preserved by countable Products?

1 - yes (Thm 30.2).
2 - yes (Thm 30.2).
L - no, not even for finite (Example 4), but $L \times C$ is L (Exercise 14).
S - yes (Exercise 14).

Images preserved?

Cont image of L is L (Exercise 11).
Cont image of S is S (Exercise 11).
Cont open image of 1 is 1 (Exercise 12).
Cont open image of 2 is 2 (Exercise 12).
Cont image of 1 need not be 1 (Counter Examples in Top., #98).
Cont image of 2 need not be 2 (Gen. Top., Willard, 16B.1).

Counter Examples

1	$\not\Rightarrow$	$2, C, L, S$	Example: $\mathbb{R}_{\text{discrete}}$
1	$\not\Rightarrow$	M	Example: $\mathbb{R}_l$
2	$\not\Rightarrow$	C	Example: $\mathbb{R}$
2	$\not\Rightarrow$	M	Example: $\mathbb{R}_{\text{trivial}}$
C	$\not\Rightarrow$	$1, 2$	Example: $\mathbb{R}_{\text{fin. comp.}}$
C	$\not\Rightarrow$	M	Example: $\mathbb{R}_{\text{trivial}}$
C	$\not\Rightarrow$	S	Example: Counter Examples in Top., #17
L	$\not\Rightarrow$	C	Example: $\mathbb{R}$
L	$\not\Rightarrow$	M	Example: $\mathbb{R}_l$
L	$\not\Rightarrow$	$1, 2$	Example: $\mathbb{R}_{\text{fin. comp.}}$
L	$\not\Rightarrow$	S	Example: Counter Examples in Top., #17
S	$\not\Rightarrow$	1	Example: $\mathbb{R}_{\text{fin. comp.}}$
S	$\not\Rightarrow$	2	Examples: $\mathbb{R}_{\text{fin. comp.}}, \mathbb{R}_l$
S	$\not\Rightarrow$	C	Example: $\mathbb{R}$
S	$\not\Rightarrow$	L	Example: $\mathbb{R}_l^2$
S	$\not\Rightarrow$	M	Examples: $\mathbb{R}_{\text{trivial}}, \mathbb{R}_{\text{fin. comp.}}, \mathbb{R}_l$