Summary of §31 & §32 Munkres

Codes: 2 = second countable, C = compact, N = normal, R = regular, H = Hausdorff, M = metrizable, LC = locally compact, L = Lindelöf.

Basic relationships.

$$
\begin{array}{rcll}
N & \Longrightarrow & R & \Longrightarrow \quad H \\
H & \not\Longrightarrow & R & \text{(Example 1, §31)} \\
R & \not\Longrightarrow & N & \text{(Example 3, §31)} \\
2\&R & \Longrightarrow & N & \text{(Theorem 32.1)} \\
M & \Longrightarrow & N & \text{(Theorem 32.2)} \\
C\&H & \Longrightarrow & N & \text{(Theorem 32.3)} \\
LC\&H & \Longrightarrow & R & \text{(Exercise 3, §32)} \\
R\&L & \Longrightarrow & N & \text{(Exercise 4, §32)}
\end{array}
$$

Inherited by Subspaces?

H - yes (Theorem 31.2).
R - yes (Theorem 31.2).
N - no (Example 1, §32), but yes for closed (Exercise 1, §32).

Preserved by products?

H - yes (Theorem 31.2).
R - yes (Theorem 31.2).
N - no, not even for finite (Example 3, §31).
If $\prod X_\alpha$ is N, R or H, then each X_α is N, R or H resp. (Exercise 2, §32).

Continuous images preserved?

None are for H, R or N. But we do have the following. Let $f : X \to Y$.
If f is cont, open, closed, and X is R, then $f(X)$ is H. (Willard, Thm 14.6)
If f is cont, closed, and X is N, then $f(X)$ is N. (Willard, Thm 15.4c)

Related categories.

Other types of topological spaces include *completely regular*, *completely normal*, *preferably normal*, *Tychonoff*, and *Urysohn* spaces. Some of these are in the exercises of Munkres and all are in Willard. Note that Willard's terminology is a bit different from Munkres. Older books and papers likely use Willard's definitions, more recent ones Munkres. Make sure you know what terminology is being used when doing reading for you research.