Summary of Chapter 6

Def A collection of subsets of X, $\mathcal{A}$, is *locally finite* if $\forall x \in X$ $\exists$ a nbhd U of x that meets only finitely many members of $\mathcal{A}$.

Def A collection of subsets of X, $\mathcal{A}$, has a *refinement* $\mathcal{B}$ if $\forall B \in \mathcal{B}$ $\exists A \in \mathcal{A}$ with $B \subset A$.

Def A top. sp. X is *paracompact* if every open cover $\mathcal{A}$ of X has a locally finite refinement $\mathcal{B}$ that covers X.

Ex $\mathbb{R}$, $\mathbb{R}^n$ are paracompact.

Thm (41.1) Paracompact and Hausdorff $\Rightarrow$ normal.

Thm (41.2) Closed subsets of paracompact spaces are paracompact.

Fact The product of two paracompact spaces need not be paracompact. $\mathbb{R}_\ell$ is paracompact, but $\mathbb{R}_\ell$ is not.

Thm 41.4 Metrizable $\Rightarrow$ paracompact.

Thm 41.5 Regular and Lindelöf $\Rightarrow$ paracompact.

Def Let $\{U_\alpha\}_{\alpha \in J}$ be an indexed open covering of X. An indexed family of cont. functions

$$\phi_\alpha : X \to [0,1]$$

is a partition of unity on X with respect to $\{U_\alpha\}$, if

(1) $\operatorname{supp} \phi_\alpha \subset U_\alpha \quad \forall \alpha$,

(2) $\{\operatorname{supp} \phi_\alpha\}$ is locally finite,

(3) $\sum \phi_\alpha(x) = 1 \quad \forall x \in X$.

Thm 41.7 Let X be paracompact and Hausdorff. Let $\{U_\alpha\}$ be an indexed open covering of X. Then $\exists$ a partition of unity on X wrt $\{U_\alpha\}$.

Thm 40.3 (Nagata-Smirnov metrization theorem)

X metrizable $\Longleftrightarrow$ X reg. and has a basis that is countably locally finite.

Thm 42.1 (Smirnov metrization theorem)

X metrizable $\Leftrightarrow$ paracompact, Hausdorff and is locally metrizable.