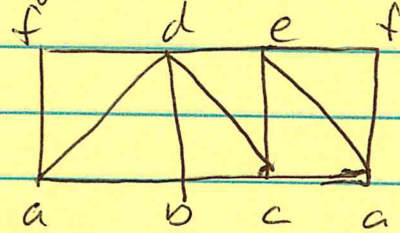
§3

Abstract Simplicial Complexes
(outline)

Def An abstract simplicial complex $\mathcal{S}$ is a collection of finite nonempty set s.t. if $A \in \mathcal{S}$ has two or more elements, then each nonempty subset of A is in $\mathcal{S}$.

Notice that simplicial complexes have this property. If we give each member of $\mathcal{S}$ an order we can define boundary maps and hence homology groups of abs. sim. com's. This allows for greater flexability

Ex Use this diagram as a shorthand for an abs. sim. com.

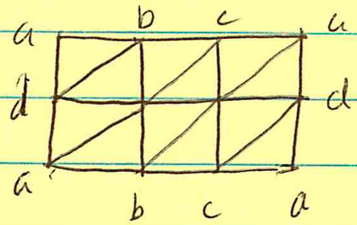


$\mathcal{S} = \{[abd], [adf], [afe], [aec], [bcd], [ced],$
$[ab], [ad], [af], [ae], [ac], [bd], [bc],$
$[cd]\ [ce], [de]\ [df], [ef], [a], [b], [c],$
$[d], [e], [f]\}$

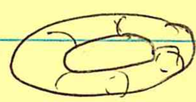
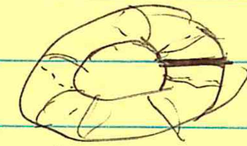
If you identify the two edges labeled $[af]$ you get a topological cylinder. Thus, we can compute its homology groups without having to construct a sim com. in $\mathbb{R}^3$.

The textbook shows (Thm 3.1) that every abs. sim. com. is isomorphic to some sim. com.

Here is an example where the labeling does not work in the way you might expect.



is not a torus!

Instead of  you get  ← pinched

Lemma 3.2 shows how to avoid such problems.