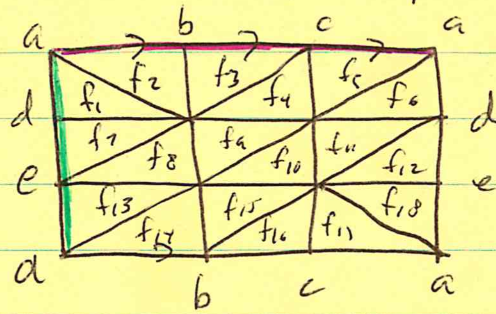
§42 continued. Surfaces (a little)

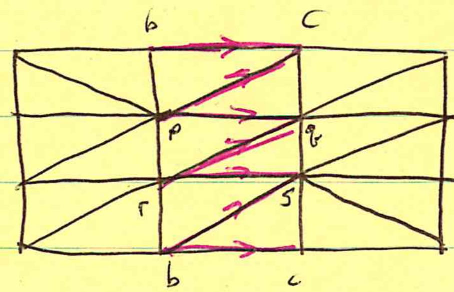
Recall when we did the simplicial homology of the torus it was clear/easy to spot generators.




all $f_i$ $\circlearrowright$).

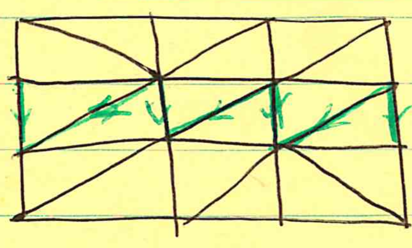
$[a,b]+[b,c]+[c,a]$ and $[a,d]+[d,e]+[e,a]$ are generators for $H_1$. But $[a,b]^*+[b,c]^*+[c,a]^*$ is not even a cocycle! Check that $\delta([a,b]^*+[b,c]^*+[c,a]^*) = -f_2^*+f_{14}^*-f_3^*+f_{16}^*-f_5^*+f_{17}^*$.

So, why do 1-cocycles look like?




Let $\omega' = [b,c]^*+[p,c]^*+[p,q]^*+[r,q]^*+[r,s]^*+[b,s]^*$

$\delta\omega' = -f_3^*+f_{16}^*+f_3^*-f_4^*+f_4^*-f_9^*+f_9^*-f_{10}^*+f_{10}^*-f_{15}^*+f_{15}^*-f_{16}^* = 0.$

Let $z' =$  $= [d,e]^*+[p,e]^*+[p,r]^*+[q,r]^*+[q,s]^*+[d,s]^*.$

Check that $\delta z' = 0$.

You should compute the following:

$$\langle w^1, w_1 \rangle = 1 \qquad \langle w^1, z_1 \rangle = 0$$
$$\langle z^1, w_1 \rangle = 0 \qquad \langle z^1, z_1 \rangle = 1.$$

where $w_1 = [a,b] + [b,c] + [c,a]$
$z_1 = [a,d] + [d,e] + [e,a]$.

We will show later that $H^1(K)$ is generated by $w^1$ and $z^1$.

Thm 42.1 If $|K|$ is connected, then $H^0(K) \cong \mathbb{Z}$.
If $|K|$ has $n$ components, then $H^0(K) \cong \mathbb{Z}^n$.

Pf see textbook.

Def We can do reduced cohom just like we did reduced homology.

$$C_1(K) \xrightarrow{\partial_1} C_0(K) \xrightarrow{\varepsilon} \mathbb{Z}$$

yields

$$\mathrm{Hom}(C_1(K), \mathbb{Z}) \xleftarrow{\delta_1} \mathrm{Hom}(C_0(K), \mathbb{Z}) \xleftarrow{\tilde{\varepsilon}} \underset{\mathrm{Hom}(\mathbb{Z}, \mathbb{Z})}{\overset{\textstyle\mathbb{Z}}{\shortparallel}}$$

or $\mathbb{Z} \xrightarrow{\tilde{\varepsilon}} C^0(K) \xrightarrow{\delta_1} C^1(K)$.

$$\tilde{H}^0(K) = \frac{\ker \delta_1}{\mathrm{im}\, \tilde{\varepsilon}}.$$

Thm 4.2.2 If $|K|$ is conn'd, then $\tilde{H}^0(K) = 0$.
In general $H^0(K) \cong \tilde{H}^0(K) \oplus \mathbb{Z}$.

pf See textbook.