Problem. Find the homology groups of $Z = S^2 \times S^3 \times S^7 \times S^7$.

Let $Y = S^2 \times S^3$. We know how to compute the homology groups of Y.

$$H_p(Y) \cong \begin{cases} \mathbb{Z} & \text{for} \quad p = 0 \\ 0 & \text{for} \quad p = 1 \\ \mathbb{Z} & \text{for} \quad p = 2 \\ \mathbb{Z} & \text{for} \quad p = 3 \\ 0 & \text{for} \quad p = 4 \\ \mathbb{Z} & \text{for} \quad p = 5 \end{cases}$$

Let $X = Y \times S^7$. Now we compute as follows.

$$\begin{array}{lllllll}
H_0(X) & \cong & \mathbb{Z} & & & & \\
H_1(X) & \cong & H_{-6}(Y) \oplus H_1(Y) & \cong & 0 \oplus 0 & = & 0 \\
H_2(X) & \cong & H_{-5}(Y) \oplus H_2(Y) & \cong & 0 \oplus \mathbb{Z} & = & \mathbb{Z} \\
H_3(X) & \cong & H_{-4}(Y) \oplus H_3(Y) & \cong & 0 \oplus \mathbb{Z} & = & \mathbb{Z} \\
H_4(X) & \cong & H_{-3}(Y) \oplus H_4(Y) & \cong & 0 \oplus 0 & = & 0 \\
H_5(X) & \cong & H_{-2}(Y) \oplus H_5(Y) & \cong & 0 \oplus \mathbb{Z} & = & \mathbb{Z} \\
H_6(X) & \cong & H_{-1}(Y) \oplus H_6(Y) & \cong & 0 \oplus 0 & = & 0 \\
H_7(X) & \cong & H_0(Y) \oplus H_7(Y) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z} \\
H_8(X) & \cong & H_1(Y) \oplus H_8(Y) & \cong & 0 \oplus 0 & = & 0 \\
H_9(X) & \cong & H_2(Y) \oplus H_9(Y) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z} \\
H_{10}(X) & \cong & H_3(Y) \oplus H_{10}(Y) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z} \\
H_{11}(X) & \cong & H_4(Y) \oplus H_{11}(Y) & \cong & 0 \oplus 0 & = & 0 \\
H_{12}(X) & \cong & H_5(Y) \oplus H_{12}(Y) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z}
\end{array}$$

We use that $Z = X \times S^7$.

$$\begin{array}{lllllll}
H_0(Z) & \cong & \mathbb{Z} & & & & \\
H_1(Z) & \cong & H_{-6}(X) \oplus H_1(X) & \cong & 0 \oplus 0 & = & 0 \\
H_2(Z) & \cong & H_{-5}(X) \oplus H_2(X) & \cong & 0 \oplus \mathbb{Z} & = & \mathbb{Z} \\
H_3(Z) & \cong & H_{-4}(X) \oplus H_3(X) & \cong & 0 \oplus \mathbb{Z} & = & \mathbb{Z} \\
H_4(Z) & \cong & H_{-3}(X) \oplus H_4(X) & \cong & 0 \oplus 0 & = & 0 \\
H_5(Z) & \cong & H_{-2}(X) \oplus H_5(X) & \cong & 0 \oplus \mathbb{Z} & = & \mathbb{Z} \\
H_6(Z) & \cong & H_{-1}(X) \oplus H_6(X) & \cong & 0 \oplus 0 & = & 0 \\
H_7(Z) & \cong & H_0(X) \oplus H_7(X) & \cong & \mathbb{Z} \oplus \mathbb{Z} & = & \mathbb{Z}^2 \\
H_8(Z) & \cong & H_1(X) \oplus H_8(X) & \cong & 0 \oplus 0 & = & 0 \\
H_9(Z) & \cong & H_2(X) \oplus H_9(X) & \cong & \mathbb{Z} \oplus \mathbb{Z} & = & \mathbb{Z}^2 \\
H_{10}(Z) & \cong & H_3(X) \oplus H_{10}(X) & \cong & \mathbb{Z} \oplus \mathbb{Z} & = & \mathbb{Z}^2 \\
H_{11}(Z) & \cong & H_4(X) \oplus H_{11}(X) & \cong & 0 \oplus 0 & = & 0 \\
H_{12}(Z) & \cong & H_5(X) \oplus H_{12}(X) & \cong & \mathbb{Z} \oplus \mathbb{Z} & = & \mathbb{Z}^2 \\
H_{13}(Z) & \cong & H_6(X) \oplus H_{13}(X) & \cong & 0 \oplus 0 & = & 0 \\
H_{14}(Z) & \cong & H_7(X) \oplus H_{14}(X) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z} \\
H_{15}(Z) & \cong & H_8(X) \oplus H_{15}(X) & \cong & 0 \oplus 0 & = & 0 \\
H_{16}(Z) & \cong & H_9(X) \oplus H_{16}(X) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z} \\
H_{17}(Z) & \cong & H_{10}(X) \oplus H_{17}(X) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z} \\
H_{18}(Z) & \cong & H_{11}(X) \oplus H_{18}(X) & \cong & 0 \oplus 0 & = & 0 \\
H_{19}(Z) & \cong & H_{12}(X) \oplus H_{19}(X) & \cong & \mathbb{Z} \oplus 0 & = & \mathbb{Z}
\end{array}$$