Topics in 3-Manifold Topology (Trefoil Surgery)

Math 532

Summer 2013

Instructor: Prof. Sullivan

Textbook: None

Prerequisite: Math 452

Course Goal: To prove Moser's theorem on Dehn surgery on torus knots and apply this to the study of flows on manifolds. Obviously there is a lot of background to cover. Homework will mainly consist of short proofs. As we get further in the course you should not expect to understand every detail but should get the "big picture". It is not realistic to give tests at this level so I won't. I will expect students to come to class and be inquisitive. Grades will be based on regular attendance and the homework.

Outline

Review basics: Open, closed, compact and connected sets, continuity.

Manifolds: Definitions, examples, classification of surfaces.

Three dimensional Manifolds: Connected sums, knots, Dehn surgery.

Homology groups: Define H1(M,Z). Examples. Finitely generated abelian groups.

Seifret Fiber Spaces: Local data, constructions, notations and classification.

Surgery Theorem: Statement and proof.

Flows: Stability, periodic orbits, Morse-Smale flows, Wada's Theorem, Smale flows, Birman-Williams theorem on templates, classification of Lorenz-Smale flows.

Here is the first paragraph of the notes will use after the background material. It should give you an idea of the flavor of the course:

Imagine you live in some sort of three-dimensional universe. But now suppose someone (or something - maybe some sort of space worm) has carved out a tunnel in space that forms a loop. Annoyed at this you grab a loop of ``space tubing'' to fill in the tunnel, gluing the outside of the tube to the wall of the tunnel. Now you go back to whatever it was you were doing before, but strange things start happening. Because you weren't careful in how you did the gluing it turns out the structure of your three dimensional world has changed. Indeed, it might not even be prime! This operation, called Dehn surgery, is a fundamental tool in the study of three-dimensional manifolds (spaces). Our goal here is to

develop this theory, somewhat informally, and then use it to prove a classical theorem due to Louise Moser that describes the types of manifolds that can be derived by performing Dehn surgery on a trefoil shaped tunnel in a standard space called the three-dimensional sphere.