Course policies for math C13-1, Winter quarter, 1996.

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Text: *A First Course in Chaotic Dynamical Systems*, by Robert Devaney, is the primary text. *An Introduction to Chaotic Dynamical Systems*, also by Robert Devaney, will be used as a supplement. It is on reserve in the Math Library (Lunt Hall).

Tests: There will a mid-term test and a final. Part of the mid-term may be take home.

If you miss a test because you were deathly ill and have a note from a doctor, or because a close relative died and you can give me the funeral home's phone number to check, then you may take a make-up test. Otherwise, forget it. Also, you must call or e-mail me in advance.

Test dates:

- Mid-Term: Tuesday, February 6, 9am-9:50, Lunt 106.
- Final: Tuesday, March 12, 9am-11am, Lunt 105.

Homework: Homework will be assigned in class and will be due on the Friday of the next week. You are encouraged to work together on the homework, but the work you turn in should be your own.

Grades: Your grade will be based on 1/3 for the mid-term, 1/3 for the final and 1/3 for the homework.

Cheating: The minimum penalty for cheating is a grade of F in the course. Your Dean may wish to take further action. This is a big no no.

TEST: C13-1, Midterm Exam, Feb. 6, 1996

1. Use the Intermediate Value Theorem to give a rigorous proof that if a continuous function on the real line has an orbit of period 2, then it must have a fixed point.

2. Let $f_\lambda(x) = \lambda x(1-x)$ and $g_c(x) = x^2 + c$. Prove that if $c < 1/4$, then there is a unique $\lambda > 1$ such that g_c is topologically conjugate to f_λ via a map of the form $h(x) = ax + b$.

3. Describe a symbol space for the function depicted below (you need three symbols).

   How many distinct orbits of period 2 are there? How many distinct orbits of period 3 are there?

4. a) Consider the function $g(x) = 1.1\cos^2(x)$ (here the 2 is a power). Use a calculator to find the attracting orbit of period two, to three decimal places. (The convergence is slow, so you'll need to be clever.) Show that this orbit really is attracting by finding the derivative. (You should use radians in your calculations.)

   b) Now, by problem 1 there is a fixed point in between the two points of period two you just found. But it is a repeller so how can we find it by numerical means? Answer this question and then find the repeller to three decimal places. Hint: look at the inverse of g.

Final exam, C13-1, Winter, 1996

1. Let $M_{01} = \{\mathbf{s} \in \Sigma | s_0 = 0, s_1 = 1\}$ and $M_{101} = \{\mathbf{s} \in \Sigma | s_0 = 1, s_1 = 0, s_2 = 1\}$.

   a) What is the minimum distance possible between a point in M_{01} and a point in M_{101} ?

   b) Give an example of two sequences, one in M_{01} the other in M_{101}, whose distance is the minimum you found.

2. Find an element of Σ whose orbit under the shift map σ comes arbitrarily close to 00000000... and to 11111111111... but which is not dense in Σ.

3. Consider the point $\mathbf{s} = 0100100010000100001...$ in Σ.

   a) Show that the orbit of $\mathbf{s}$ under sigma comes arbitrarily close to 00000000000.....

   b) Show that the orbit of $\mathbf{s}$ under sigma comes arbitrarily close to 10000000000000.....

   c) Show that the orbit of $\mathbf{s}$ under sigma is not dense in Σ.

4. Consider the function whose graph is below. Prove it has no closed orbits with odd period greater than one. Does it have an orbit of period 6? If so, what does that imply?

5. Let X be a metric space with metric d. Let $\alpha > 0$ (a real number) and let p, q, r, and s be elements of X.

a) Suppose $d(p, r) > \alpha$, $d(p, q) < \alpha/8$ and $d(r, s) < \alpha/8$. Use the triangle equality to show that $d(q, s) > 3\alpha/4$.

b) Suppose instead that $d(p, r) < \alpha$ with $d(p, q)$ and $d(r, s)$ still less than $\alpha/8$. Find an **upper bound** for $d(q, s)$.