Summary of Calculus I (Math 150)

- Calculus is the mathematical study of continuous change.
- Limits are a way to analyze the behavior of a function near a point or as the input variable grows in magnitude without bound.
- A function $f(x)$ is **continuous** at $x = c$ if and only if the limits as x approaches c from both the left and the right exist and are equal to $f(c)$ (which must be defined).
- The derivative of a function gives the **instantaneous rate of change** or the **velocity** at any given point where the derivative is defined. It is defined using limits. The derivative can also be viewed as the slope of the line tangent to the graph at the given point. Tangent lines are used to approximate curves near some given point.
- Derivatives are also used to find **local minimums** and **maximums**. They also tell us where a function is increasing or decreasing. The second derivative tells us the **concavity** of a function which is a way to visualize acceleration. An **inflection point** is where the concavity switches sign.
- There are several techniques for computing derivatives. Probably the most important is the **Chain Rule**. It says, in essence, when you compose two functions the rates of change multiply: if Sue can run twice as fast as Bill, and Bill can run three times as fact as Doug, then Sue can run six times as fast as Doug.
- The **Mean Value Theorem** is an important tool although this may not be apparent to you now. It says, in essence, if Grand Ma is a 100 miles away and you get there in one hour, she knows you were speeding!
- **Integrals** are another application of limits, in this case the limit of a **Riemann sum**. They are used to compute areas, volumes and other physical properties like pressure.
- The **anti-derivative** turns out to be a key tool in computing integrals. The **Fundamental Theorem of Calculus** says, in essence, that the derivative and the integral are inverses of each other. The substitution trick is really just applying the Chain Rule backwards.
- Motion is change in position. This is where calculus was first applied. Letting a stand for acceleration, v for velocity, and p for position, we have

  $$a(t) = v'(t) = p''(t)$$

  and conversely,

  $$p(t) + C_1 = \int v(t)\,dt \qquad v(t) + C_2 = \int a(t)\,dt$$

  where the constants are determined by initial conditions. But calculus is used to study many other phenomena that involve continuous change.