Part I: Calculations

1. [20 points] Find the inverse of \[
\begin{bmatrix}
3 & 0 & 0 \\
0 & 4 & -1 \\
0 & 2 & 0
\end{bmatrix}
\]. Show all steps.

2. [20 points] Find all values of \(x\) such that \[
\begin{bmatrix}
1 & 2 & 1 \\
2 & 4 & x \\
1 & 3 & 2
\end{bmatrix}
\] is singular.
3. [20 points] Find an equation for the plane in $\mathbb{R}^3$ that contains the point $(1,1,1)$ and the $y$-axis.

4. [20 points] Find the solution of the initial value problem $v'(t) = Av(t)$ where

$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}, \quad v = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \quad \text{and} \quad v(0) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}.$$
5. [20 points] Apply the Gram Schmidt process to the set \( \left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 0 \end{bmatrix} \right\} \).

6. [20 points] Find the LU decomposition of \( \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & -1 \end{bmatrix} \).
7. [20 points] Find the coordinates of \[
\begin{bmatrix}
2 \\
-2 \\
2
\end{bmatrix}
\] with respect to the basis \(\left\{ \begin{bmatrix}
1 \\
2 \\
0
\end{bmatrix}, \begin{bmatrix}
0 \\
1 \\
1
\end{bmatrix}, \begin{bmatrix}
1 \\
0 \\
2
\end{bmatrix} \right\} \).

8. [20 points] Find the equation of the quadratic function that passes through the three points, \((-1, 1), (1, 2), \text{ and } (2, 4)\).
9. [10 points] Define the following terms:
   a) Linear independence.
   b) Rank of a matrix.
   c) Basis of vector space.
   d) Eigenvalue and eigenvector.
   e) Null space of a matrix.

10. [10 points] Let $A$ be an $m \times n$ matrix with rank $n$. Prove that $A^T A$ is nonsingular.
11. [10 points] Let $A$ and $B$ be $2 \times 2$ matrices. Is it possible for $AB - BA$ to be equal to the identity matrix? Prove your claim.

12. [10 points] Let $A$ be a nonsingular matrix. Suppose that $\lambda$ is an eigenvalue of $A$. Prove that $1/\lambda$ is an eigenvalue of $A^{-1}$. 