1. Let $B$ and $C$ be $2 \times 2$ matrices, and let $O$ be the $2 \times 2$ matrix with all 0 entries. Let $A$ be the $4 \times 4$ matrix given by

$$A = \begin{bmatrix} B & O \\ O & C \end{bmatrix}.$$ 

Prove that $\det(A) = \det(B) \det(C)$.

2. Let $A$ be an $n \times n$ matrix. Suppose that $A$ is skew symmetric (this means $A^T = -A$). Prove that if $n$ is odd, then $\det(A) = 0$. Give an example of a $4 \times 4$ skew symmetric matrix that has determinant 12.

3. Let $A$, $B$ and $C$ be a square matrices of the same size. Let $A \sim B$ mean $A$ is similar to $B$. Prove each of the following statements.

(a) $A$ is similar to itself.
(b) If $A \sim B$ then $B \sim A$.
(c) If $A \sim B$ and $B \sim C$ then $A \sim C$.

(This means that similarity gives an equivalence relation on the set of $n \times n$ matrices for any $n$.)