Name: ____________________________ ID #: __________________________

**NO CALCULATORS**

1. [15 points] [Review] Find the arc length of the arc of the parabola $y = x^2$ between the points $(0, 0)$ and $(1, 1)$. Hint: You will need to use $\int \sec^3 \theta \, d\theta = (\sec \theta \tan \theta + \ln(|\sec \theta + \tan \theta|))/2 + C$. If you can derive this you will get 5 points extra credit.

2. [20 points] Evaluate each series or show that it diverges.

   a) $\sum_{n=0}^{\infty} \left[ \left( \frac{3}{4} \right)^n - 2 \left( \frac{5}{6} \right)^n \right]$

   b) $\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$
3. [15 points] [Theory] Let $a$ be any constant and suppose that $|r| < 1$. Prove that the geometric series $\sum_{n=0}^{\infty} ar^n$ converges and that its value is $\frac{a}{1 - r}$.

b) Now suppose $|r| > 1$. What happens to the series and why?

c) What happens to the series when $r = \pm 1$. Explain why.
4. [30 points] For each series determine whether it converges or diverges. Justify your answer.

a) \[ \sum_{n=2}^{\infty} \frac{1}{n \ln(n)} \]

b) \[ \sum_{n=2}^{\infty} \left( \frac{\ln(1 + n^2)}{n} \right)^n \]

c) \[ \sum_{n=0}^{\infty} \frac{1}{3 + 3^n} \]
5. [30 points] For each series determine whether it converges absolutely, converges conditionally, or diverges. Justify your answer.

a) \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(n^5)} \)

b) \( \sum_{n=0}^{\infty} (-1)^n \frac{n^2 + 1}{n^7 + n^2 + 1} \)

c) \( \sum_{n=0}^{\infty} (-2)^{n+1} \frac{7}{(\sqrt{3})^n} \)