1. [20 points] For each series below compute it or show the series diverges.

a) \[ \sum_{n=3}^{\infty} \frac{1}{(n+1)(n+2)} \]

b) \[ \sum_{n=6}^{\infty} \frac{3^n + 4^n}{5^n} \]
2. [20 points] Do the series below converge or diverge? Show all steps.

a) \( \sum_{n=0}^{\infty} \frac{2n + 1}{\sqrt{n^3 + 1}} \)

b) \( \sum_{n=2}^{\infty} \frac{1}{(\ln n)^n} \)

3. [20 points] Do the series below converge absolutely, converge conditionally, or diverge? Show all steps.

a) \( \sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n} \)

b) \( \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n(n+1)}} \)
4. [20 points] Find the interval of convergence for each of the power series below. Be sure to check any end points.

\[ a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left( x + \frac{1}{3} \right)^n \]

\[ b) \sum_{n=1}^{\infty} n^2 x^n \]
5. [10 points] Find the Taylor series of \( f(x) = \frac{\sin(x) - x}{x^2} \), centered about \( x = 0 \). Express your answer in \( \Sigma \) notation.

6. [10 points] Find the first 3 terms of the Taylor series of \( \tan x \), centered about \( x = \pi/4 \).
7. [BONUS: 7+3 points]

(a) Show that \( \int_0^b h \sin^2 \left( \frac{x\pi}{b} \right) \, dx = \frac{1}{2} bh. \)

(b) Interpret this result geometrically. Hint: graph the integrand over \([0, b]\), and think about what \(b\) and \(h\) might stand for.