Review Sheet for Calculus III

On Friday there will be a quiz covering arithmetic, algebra, geometry, trigonometry and of course basic calculus. The quiz will count as a homework set. I will ask to meet individually with students who score poorly. Below are some practice problems. But these are not the quiz questions. If a problem is difficult for you, it means you need to review that topic area.

NO CALCULATORS

(1) Compute: (a) \(23.56 \times 7.12\). (b) \(\frac{17}{34} - \frac{14}{15}\) (c) \(\log_4 32\) (d) 200% of 87.

(2) Simplify: (a) \(\frac{2x + y}{x - y} + \frac{4x + y}{x + y}\) (b) \(1024x^8w^2v^{1/3} \left(\frac{x^2p^{44}v^{1/3}}{xy^{-38^2pw}}\right)^2\)

(3) A tetrahedron has edge length of 1 meter. Find its volume and surface area. Generalize.

(4) A triangle has edge lengths 3, 7 & 8. Find all the angles in radians.

(5) Compute \(\sec^{-1} \sqrt{2}\) in radians.

(6) Graph: (a) \(y = 3 \cos(2x + \pi)\) (b) \(y = e^{-x^2} \cos(x)\) (c) \(y = (x + 2)(x - 5)|x - 1|\) (d) \(y = \sin(x)/x\) (e) \(y = \sin(1/x)\).

(7) (a) If \(f(x)\) is an odd function and \(g(x)\) is an even function what can you say about \(f(f(g(f(x))))\)? (b) Can you give an example of a function that is both odd and even?

(8) Prove that \((f(x) + g(x))' = f'(x) + g'(x)\), where \(f\) and \(g\) are real valued differentiable functions of a real variable.

(9) Compute: \(\lim_{x \to -2} \frac{x^2 + 5x + 6}{x + 2}\), with and without using L’Hôpital’s Rule.
(10) Compute: \( \lim_{x \to \infty} \frac{x^2 + \sin x}{x^2} \).

(11) Let \( f(x, y) = \sin(yx^2) \frac{yx + \ln x}{yx \tan 3x} \). Assume \( x \) and \( y \) are independent variables. First compute the derivative of \( f \) with respect to \( x \). Then compute the derivative of \( f \) with respect to \( y \).

(12) \( \int_{-15}^{15} \sin x^3 \, dx \)

(13) \( \int x \cos 4x \, dx \)

(14) \( \int \sin 2\alpha \cos 7\alpha \, d\alpha \)

(15) \( \int \arctan 3P \, dP \)

(16) \( \int xe^x + xe^{x^2} \, dx \)

(17) \( \int \frac{x}{\sqrt{9 - x^2}} \, dx \)

(18) Consider the region formed from rotating the portion of the parabola given by \( y = 4 - x^2 \) for \( y \geq 0 \) about the \( y \)-axis. Find its volume and surface area not including the base.

(19) Derive the formula for the volume of a sphere as a function of its radius \( R \). Take its derivative. What does this give you?

(e) Why is \( e \) important? Give the value of \( e \) to five decimal places. Find an infinite series that converges to \( e \).