A continuum is a connected, compact, metric space. A continuum is non-degenerate if it has more than one point. In the plane a closed arc, a circle, and the topologist’s sine curve are examples. A topological space \( X \) is homogeneous if for every \( a, b \in X \), there is a homeomorphism, \( h : X \rightarrow X \) with \( h(a) = b \). From the above examples only the circle is homogeneous.

In 1920 Knaster and Kuratowski [4] asked whether the circle was the only non-degenerate homogeneous continuum in the plane. In 1948 Bing [1] showed that the pseudo-arc was another such space and in 1959 Bing and Jones [2] showed that the circle of pseudo-arcs was a third such space. Any two pseudo-arcs or circles of pseudo-arcs are respectively homeomorphic, just as any two simple closed curves are. The circle, the pseudo-arc and the circle of pseudo-arcs are topologically distinct spaces.

In 2014 Hoehn and Oversteegen [3] showed that up to homeomorphism these are the only three examples, thus giving a complete topological classification of non-degenerate, homogeneous, planar continua.

Here is a construction of the pseudo-arc. All sets are in the plane. A \( \epsilon \)-chain is an ordered finite collection of open sets, \( C = (L_1, L_2, \ldots, L_n) \), such that each link of \( C \) is a subset of some link of \( C' \). We say \( C \) is crooked in \( C' \) if for all indices \( i, j, m \) and \( n \) with \( L_i \cap L'_m \neq \emptyset \), \( L_j \cap L'_n \neq \emptyset \) and \( m < n - 2 \), there exists indices \( k \) and \( l \) such that \( i < k < l < j \) or \( i > k > l > j \), \( L_k \subset L'_{n-1} \), and \( L_l \subset L'_{m+1} \).

A pseudo-arc is then constructed as follows. Let \( p, q \in \mathbb{R}^2 \), \( p \neq q \). For \( i = 1, 2, 3 \ldots \) let \( C^i = (L^i_1, \ldots, L^i_{n_i}) \) be \( \epsilon \)-chains such that

- \( p \in L^i_1 \) and \( q \in L^i_{n_i} \),
- \( \epsilon \)-chain with \( \epsilon = 1/2^i \),
- the closure of each link of \( C^{i+1} \) is a subset of some link of \( C^i \), and
- \( C^{i+1} \) is crooked in \( C^i \).
Let \( P = \bigcap_{i=1}^{\infty} \left( \bigcup_{j=1}^{n_i} L_i^j \right) \). Then \( P \) is a pseudo-arc.

**Remark.** A similar definition can be given for the **pseudo-circle** (each last link meets the first and \( p = q \)). However, is has been shown that the pseudo-circle is not homogeneous. [5]

Finally we discuss the circle of pseudo-arcs. A space \( X \) is a circle of pseudo-arcs if there exists an open continuous function \( f : X \to S^1 \) such that \( f^{-1}(\theta) \approx P \) for every \( \theta \in S^1 \). Bing and Jones [2] showed these have the required properties and constructed a circle of pseudo-arcs in the plane.

**References**

   http://projecteuclid.org/euclid.dmj/1077475025
   http://arxiv.org/abs/1409.6324
   http://pldml.icm.edu.pl/pldml/element/bwmeta1.element.bwnjournal-article-fmv1i1p223bwm