KNOT THEORY

- Who: Mike Sullivan.
- When: Spring 1998.
- Prerequisite: Undergraduate abstract algebra (groups, rings).
- Text: *Knots and Surfaces*, by Gilbert & Porter, Oxford Science.

Description: A knot is an embedding of a closed loop into a three dimensional space, such as R^3. See the figures below. There three knots are shown, K_1, K_2, and K_3. At first glance K_2 and K_3 look the same. But in fact K_2 can be deformed into K_1. Try to show this. I claim K_3 is different from K_1. But how can we be sure? Couldn't there be some highly complex procedure that would transform K_3 into K_1?

Here is another problem. Take the mirror images of K_1 and K_3; call these knots $\tilde{K}_1$ and $\tilde{K}_3$ respectively. I claim that $\tilde{K}_3$ can be deformed into K_3 (try to show this!), but that $\tilde{K}_1$ is distinct from K_1. But how could one prove such a statement, and what does group theory have to do with it?

Knot theory owes its origins to Gauss and Kelvin. Gauss was interested in the magnetic field produced by an electrical current flowing through a knotted loop of wire. Kelvin thought that atoms might be knotted vortices of "either". From these applied considerations knot theory became a vibrant branch of pure mathematics. It has strong connections to the problem of classifying 3 and 4 dimensional manifolds. In the last two decades knot theory has moved back into applied fields like dynamical systems (my area), statistical mechanics, and biology (e.g. the topology of DNA).

This course will show how to solve basic questions in knot theory with both visual (topological) and algebraic techniques. The former include theorems on braiding and connected sums. The later will include the Jones polynomial and the fundamental group.