Summary of Calculus I (Math 150)

- Calculus is the mathematical study of continuous change.
- Limits are a way to analyze the behavior of a function near a point or as the input variable grows in magnitude without bound.
- A function $f(x)$ is continuous at $x = c$ if and only if the limits as x approaches c from both the left and the right exist and are equal to $f(c)$ (which must be defined).
- The derivative of a function gives the instantaneous rate of change at any given point where the derivative is defined. It is defined using limits. The derivative can also be viewed as the slope of the line tangent to the graph at the given point. Tangent lines can be used to approximate a function near the given point.
- Derivatives are also used to find local minimums and maximums. They also tell us where a function is increasing or decreasing. The second derivative tells us the concavity of a function. An inflection point is where the concavity switches sign.
- There are several techniques for computing derivatives. Probably the most important is the Chain Rule. It says, in essence, when you compose two functions the rates of change multiply: if Sue can run twice as fast as Bill, and Bill can run three times as fact as Doug, then Sue can run six times as fast as Doug.
- The Mean Value Theorem is an important tool although this may not be apparent to you now. It says, in essence, if Grand Ma is a 100 miles away and you get there in one hour, she knows you were speeding!
- Integrals are another application of limits, in this case the limit of a Riemann sum. They are used to compute areas and also volumes.
- The anti-derivative turns out to be a key tool in computing integrals. The Fundamental Theorem of Calculus says, in essence, that the derivative and the integral are inverses of each other. The substitution trick is really just applying the Chain Rule backwards.
- Motion is change in position. This is where calculus was first applied. Letting a stand for acceleration, v for velocity, and p for position, we have

  $$a(t) = v'(t) = p''(t)$$

  and conversely,

  $$p(t) + C_1 = \int v(t)\, dt \qquad v(t) + C_2 = \int a(t)\, dt$$

  where the constants are determined by initial conditions. But calculus is used to study many other phenomena that involve continuous change.